Physics 263A
Problems due April 22

1. A particle of mass m moves in one dimension, subject to a Hamiltonian $H = H_0 + W$, where H_0 is the usual harmonic-oscillator Hamiltonian $\frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2$, and where W satisfies

$W|\phi_0\rangle = 0;$
$W|\phi_1\rangle = \frac{1}{2}\hbar\omega|\phi_2\rangle;$
$W|\phi_2\rangle = \frac{1}{2}\hbar\omega|\phi_1\rangle;$
$W|\phi_n\rangle = 0 \quad for\ n > 2.$

Here the $|\phi_i\rangle$ are the (usual) eigenstates of H_0 .
Note that the constant ω which appears in the definition of W is the same ω which appears in the expression for H_0 .

Find the three lowest energies, and the corresponding states (in terms of the $|\phi_i\rangle$.

2. Consider two coupled oscillators (as discussed in complement H_V) where the system is in its ground state. Calculate the expectation value of $(X_1)^2$ (in terms of m, ω, a, and λ.)